Broadening of the Berezinskii-Kosterlitz-Thouless superconducting transition by inhomogeneity and finite-size effects

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We discuss the crucial role played by finite-size effects and inhomogeneity on the Berezinskii-Kosterlitz-Thouless (BKT) transition in two-dimensional superconductors. In particular, we focus on the temperature dependence of the resistivity, that is dominated by superconducting fluctuations above the BKT transition temperature T_{BKT} and by inhomogeneity below it. By means of a renormalization-group approach we establish a direct correspondence between the parameter values used to describe the BKT fluctuation regime and the distance between T_{BKT} and the mean-field Ginzburg-Landau transition temperature. Below T_{BKT} a resistive tail arises due to finite-size effects and inhomogeneity, that reflects also on the temperature dependence of the superfluid density. We apply our results to recent experimental data in superconducting $LaAlO_3/SrTiO_3$ heterostructures, and we extract several informations on the microscopic properties of the system from our BKT fitting parameters. Finally, we compare our approach to recent data analysis presented in the literature, where the physical meaning of the parameter values in the BKT formulas has been often overlooked.

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I. INTRODUCTION

In the last years a renewed interest emerged in the superconducting transition in two-dimensional (2D) systems, prompted by the experimental achievement of high-quality ultrathin films of superconducting materials. To this category belong both few-unit-cell thick films of layered cuprate superconductors[1–3] and the nanometer-thick layers of superconducting electron systems formed at the interface between insulating oxides in artificial $LaAlO_3/SrTiO_3$ heterostructures.[4,5] At the same time, the experimental progresses made in the last decade of intense investigation in high-temperature superconductors prompted additional measurements in thin films of conventional superconductors by means of different techniques or higher resolution than the ones available in the past. Typical examples are provided by the finite-frequency study of the optical magnetoconductivity in films of InO_x,[6] Nernst-effect measurements in amorphous films of NbSi (Ref. 7) and scanning tunneling microscopy in TiN films.[8,9]

Due to the 2D nature of these systems, the superconducting (SC) transition is expected to belong to the Berezinskii-Kosterlitz-Thouless (BKT) universality class,[10–12] where the gauge symmetry is unbroken in the SC state but the system has a finite superfluid density, which is destroyed at T_{BKT} by proliferation of vortex-antivortex phase fluctuations. As it has been discussed at length in the past literature, the BKT transition has, in principle, very specific signatures.[13,14] For example, by approaching the transition from below, the superfluid density n_s is expected to go to zero discontinuously at the BKT temperature T_{BKT}, with an "universal" relation between $n_s(T_{\mathrm{BKT}})$ and T_{BKT} itself.[13–15] Approaching instead the transition from above one has, in principle, the possibility to identify the BKT transition from the temperature dependence of the superconducting fluctuations. Indeed, in 2D the temperature dependence of several physical quantities (such as the paraconductivity or the diamagnetism) is encoded in the temperature dependence of the superconducting correlation length $\xi(T)$, that diverges exponentially at T_{BKT}, in contrast to the power law expected within Ginzburg-Landau (GL) theory.[16] A possible interpolation scheme between standard GL fluctuations and BKT phase fluctuations of the SC order parameter was proposed long ago in a seminal paper by Halperin and Nelson[17] (HN).

In the attempts made in the past to find out experimental signatures of the BKT transition in thin films of conventional superconductors[18–20] it turned out that BKT fluctuations are usually restricted to a small temperature regime near the GL transition temperature T_c. If the energy range T_c-T_{BKT} is extremely small, most of the fluctuation regime is dominated by GL fluctuations, and the exponential signatures of BKT fluctuations can be hardly detected. The predominance of the GL fluctuation regime has been confirmed also by more recent measurements of Nernst effect in NbSi films[7] and zero-bias tunneling conductance in TiN films.[9] Analogously, the expected universal jump of $n_s(T_{\mathrm{BKT}})$ at T_{BKT} due to vortex proliferation can be overscreened by the simultaneous fast decrease in $n_s(T)$ due to quasiparticle excitations near T_c.[20]

An additional effect that can mask the occurrence of BKT transition is the intrinsic inhomogeneity of the sample. For example, as it has been discussed recently in the context of thin films of high-temperature superconductors,[21] the spatial inhomogeneity can broaden considerably the universal jump of the superfluid density, leading to a smooth downturn of the superfluid density instead of the sharp one expected in ultrathin samples. Such an intrinsic mesoscopic inhomogeneity has been revealed by scanning tunneling spectroscopy in cuprate superconductors,[22] and recently also in films of conventional superconducors.[8] This indicates that inhomogeneity is a crucial ingredient to several superconducting systems in the presence of disorder, as suggested also by recent numerical simulations.[23] Finally, one must account also for finite-size effects (at the scale of the sample dimensions or even smaller, in the inhomogeneous case), that are expected to cut off the divergence of the correlation length at T_{BKT}.

In this work we aim to address the role of inhomogeneity and finite-size effects in the BKT transition, providing a general scheme for analyzing paraconductivity measurements in quasi-2D superconductors. We first match the HN interpolation scheme with a detailed analysis of the renormalization-group (RG) equations for the BKT transition far from the critical region where an analytical solution is available,[12] taking into account finite-size effects. This analysis allows us to relate the parameters of the standard BKT correlation-length expression to microscopic quantities, reducing considerably the degrees of freedom in the fitting procedure of the resistivity data. In particular, we show that once fixed the difference T_c-T_{BKT} and the value of the vortex-core energy, whose role has been recently discussed in the context of the physics of high-temperature superconductors,[21,24–27] the behavior of the resistivity from T_{BKT} up to temperatures far above T_c is uniquely determined. In this way we establish a consistency check that the fitting parameters must satisfy when the standard BKT approximated formulas are used, a fact that has been often overlooked in the literature. Building on such an analysis we can also take into account the role of inhomogeneities, which as we will show are crucial to understand the experimental situation. Indeed even relatively small inhomogeneities can provide a significant enhancement of the resistive response below the T_{BKT} transition, that is simultaneously reflected in the temperature dependence of the superfluid density across the transition itself. A paradigmatic example of application of our analysis is provided by recent measurements in superconducting heterostructures.[4,5] As we shall see, a correct treatment of finite-size effects and inhomogeneity allows us to reproduce with great accuracy the available experimental results, and to estimate the superfluid density in these unconventional systems.

We note that although the BKT transition has been already invoked to discuss the physics of such heterostructures, a very different approach was taken so far in the literature,[4,5,28] relying essentially on the idea that the GL temperature T_c is far larger than T_{BKT} (that can have eventually a different qualitative character[4]) so that the *whole fluctuation regime* can be described within BKT theory. A schematic view of the difference between our analysis and the previous approach is shown in Fig. 1. The point of view taken in Refs. 4, 5, and 28 leads of course to a very different identification of the various temperature regimes, and very different physical parameters for the underlying BKT theory. In our view these previous analyses suffer from several problems, that we will discuss explicitly in this paper. In particular, they do not correctly identify the physical parameters of the system and thus overlook several interesting consequences that one can extract from the accurate experimental measurements performed in these new materials.

The structure of the paper is the following. In Sec. II we review the standard description of fluctuation conductivity in 2D superconductors, that establishes the correspondence between resistivity and the fluctuation correlation length. In Sec. III we analyze systematically the behavior of the correlation length within BKT theory, by means of a RG approach. This allows us to fully identify the parameters in the HN formula interpolating between the BKT transition and the standard Gaussian fluctuations. Section IV is devoted to the discussion of the role of inhomogeneity, through the direct application of our interpolating BKT-to-GL scheme to the resistive transition. We apply our scheme to identify the relevant parameters in superconducting heterostructures. In Sec. V we clarify the differences between our approach and previous theoretical reports. The reader interested only to the issue of analyzing the experimental data can refer directly to the Secs. IV and V. Finally, Sec. VI contains the concluding remarks.

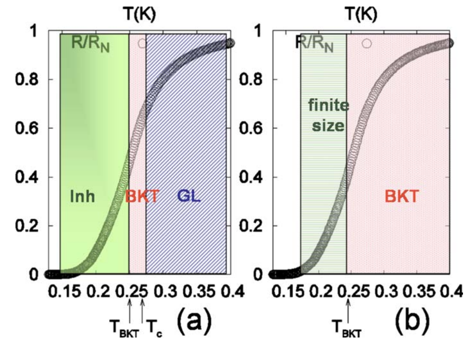

FIG. 1. (Color online) Comparison between the approach discussed in our paper and the one presented so far in the literature (Refs. 4, 5, and 28) as far as the resistivity data in superconducting heterostructures are concerned. The experimental data for the resistivity R normalized to the normal-state value R_N are taken from Ref. 5, and are the same as the ones showed in Figs. 7 and 9 below. On the left panel we summarize our approach; most of the fluctuation regime above T_{BKT} is dominated by GL fluctuations while BKT fluctuations are restricted to a narrow range of temperatures near the transition temperature T_{BKT} that would be observable in the homogeneous system. However, inhomogeneity leads to a considerable tail in the resistivity, that remains finite below T_{BKT}. On the right panel we summarize the point of view used in Refs. 4, 5, and 28; the whole range of temperatures above T_{BKT} is dominated by SC fluctuations having BKT character, and finite-size effects are responsible for the resistive tail below T_{BKT}.

II. FLUCTUATION CONDUCTIVITY IN 2D SUPERCONDUCTORS

The definition of fluctuation conductivity within the BKT theory relies on the Bardeen-Stephen formula,[29] which gives the excess conductivity $\Delta\sigma \equiv \sigma-\sigma_N$ with respect to the normal-state conductivity σ_N as a function of the density of free vortices n_F above T_{BKT} as

$$\Delta\sigma = \frac{e^2}{\hbar^2\pi^2\mu_V}\frac{1}{n_F}, \quad (1)$$

where the vortex mobility is $\mu_V=2\pi\xi_0^2c^2\rho_n/\Phi_0^2$, ξ_0 is the spacing for the vortex lattice, that we will assume equal to the zero-temperature coherence length, ρ_n is the normal-state resistivity, and Φ_0 is the flux quantum. The vortex density is also conventionally defined in terms of the correlation length ξ as $2\pi n_F \equiv 1/\xi^2$ so that near the transition where $\Delta\sigma \gg \sigma_N$

the ratio between the resistance R and its normal-state value R_N is

$$\frac{R}{R_N} = 2\pi\xi_0^2 n_F = \left(\frac{\xi_0}{\xi}\right)^2, \quad (2)$$

which is the formula usually quoted in the literature for the paraconductivity due to vortices, once that $\xi(T)$ is calculated within BKT theory. Remarkably, in 2D the same formula is valid for the Aslamazov-Larkin (AL) contribution of GL fluctuations.[16,30] However, in this case the temperature dependence of $\xi(T)$ follows $\xi_{\rm GL}^2 \sim 1/\log(T/T_c)$, or equivalently $\xi_{\rm GL}^2 \sim T_c/(T-T_c)$, where T_c is the GL or mean-field critical temperature. In 2D films the true transition occurs at a $T_{\rm BKT}$ lower than T_c, the distance between the two being a function of the microscopic parameters of the system. Thus, the fluctuation conductivity crosses over from a GL regime, where it shows a tendency to a power-law divergence at T_c, to a KT regime, where the correlation length diverges asymptotically when $T \to T_{\rm BKT}$ as $\xi \sim \exp(b/\sqrt{t})$, where we introduced the reduced temperatures

$$t \equiv \frac{T - T_{\rm BKT}}{T_{\rm BKT}}, \quad t_c \equiv \frac{T_c - T_{\rm BKT}}{T_{\rm BKT}}. \quad (3)$$

As it was observed already by Halperin and Nelson,[17] the BKT fluctuations are expected to be present only in the range of temperatures $t \ll t_c$. Thus, using the general expression $\Delta\sigma \propto \xi^2$, they proposed a well-known interpolation formula between the GL and BKT regimes given by

$$\Delta\sigma = a_{\rm HN}\sigma_N \sinh^2(\sqrt{b_{HN} t_c/t}), \quad (4)$$

where $b_{\rm HN}$ is a dimensionless constant of order 1. Thus, for $b_{\rm HN} t_c/t \gg 1$ one recognizes the exponential divergence characteristic of the BKT theory while for $b_{\rm HN} t_c/t \ll 1$ one recovers a power-law increase typical of GL fluctuations. For the prefactor $a_{\rm HN}$ Halperin and Nelson assumed $a_{\rm HN}=0.37/b_{\rm HN}$, following the Beasley-Mooij-Orlando[31] approximate relation between σ_N and the ratio $T_{\rm BKT}/T_c$ in dirty BCS superconductors. We note in passing that in the dirty BCS limit one can indeed consider only the AL contribution to the GL paraconductivity while in clean samples also the Maki-Thomson contribution can be sizable, making the analysis of GL fluctuations more involved.[19]

In the HN formula some ambiguity is still present in the choice of the parameters, i.e., the prefactor and the exponential coefficient. In this work we want to fix this ambiguity by determining exactly their values from a RG analysis of the BKT correlation length. In this way, we provide a clear procedure to analyze experimental data by respecting the internal consistency between the fitting parameters for the paraconductivity. Moreover, we will discuss how finite-size effects and inhomogeneity affect the fluctuations resistivity above and below $T_{\rm BKT}$. All these issues turn out to be crucial to correctly interpret the experimental data, as we will discuss in Secs. IV and V.

III. BEHAVIOR OF THE CORRELATION LENGTH WITHIN THE BKT THEORY

As it was shown by Kosterlitz in its original work,[12,13] the critical properties of the BKT transition can be captured by the analysis of the RG equations for the two main quantities involved in the SC transition: the superfluid stiffness J and the vortex fugacity $g=2\pi e^{-\beta\mu(T)}(\beta=1/k_BT)$, where μ is the vortex-core energy. The stiffness J is the energy scale associated to the 2D superfluid density $n_s^{\rm 2D}$ (measured experimentally via the inverse penetration depth λ), including already the temperature depletion due to quasiparticle excitations,

$$J(T) = \frac{\hbar^2 n_s^{\rm 2D}(T)}{4m^* k_B} = \frac{\hbar^2 c^2}{16\pi e^2}\frac{d}{\lambda^2(T)}, \quad (5)$$

where d is the film thickness and m^* is the effective mass of the carriers. The vortex-core energy μ is in general a multiple of the stiffness itself,[21,24,32]

$$\mu(T) = \tilde{\mu} J(T), \quad (6)$$

where $\tilde{\mu}$ is a dimensionless constant. As we discussed recently,[21,24,25] a general approach to the BKT transition can require to assume that $\tilde{\mu}$ deviates with respect to the conventional value $\tilde{\mu}_{\rm XY}=\pi^2/2$ that it acquires in the XY model[12,13,32] so that we shall use

$$\tilde{\mu} = \alpha\tilde{\mu}_{\rm XY} = \alpha\frac{\pi^2}{2}. \quad (7)$$

The RG equations in the variables g and $K=\pi J_s/T$ can be written (in analogy with the notation used for the sine-Gordon model[14]) as

$$\frac{dK}{d\ell} = -K^2 g^2, \quad (8)$$

$$\frac{dg}{d\ell} = (2-K)g, \quad (9)$$

where $\ell = \ln a/\xi_0$ and ξ_0, a are the original and rescaled RG lattice spacing, respectively. The physical value of the superfluid density J_s is determined by the limiting value of K under RG flow, i.e., $J_s \equiv TK(\ell\to\infty)/\pi$. In the low-temperature regime $K>2$ so that the vortex fugacity scales to zero under RG flow and J_s is finite, with a small renormalization with respect to the initial value. Instead at high temperature $K<2$ the vortex fugacity becomes relevant, it diverges under RG flow and as a consequence J_s scales to zero. The BKT temperature is the one where the above system of equations reaches the fixed point $K=2$, $g=0$ so that at $T_{\rm BKT}$,

$$\frac{\pi J_s(T_{\rm BKT})}{T_{\rm BKT}} = 2, \quad (10)$$

i.e., one recovers the universal jump of the superfluid density.[13–15]

The usual definition[12,13] of the correlation length ξ above $T_{\rm BKT}$ relies on the determination in the RG Eqs. (8) and (9) of a characteristic scale $\bar{\ell}$ at which the vortex fugacity is "sufficiently large." The exact definition of this scale is somehow arbitrary but it does not change qualitatively the results. In practice, we shall use as a working definition the scale ℓ_s, where the RG parameter $K(\ell_s)$ related to the superfluid density vanishes. The vortex density and correlation length are then defined as

$$2\pi n_F = \frac{g(\bar{\ell})}{2\pi a^2(\bar{\ell})} = \frac{1}{\xi^2}, \quad a(\bar{\ell}) = \xi_0 e^{\bar{\ell}}, \tag{11}$$

where $\bar{\ell}=\min(\ell_s,\ell_{\max})$, where $\ell_{\max}$ takes into account finite-size effects, as we shall discuss below. Near $T_{\rm BKT}$ one expects to recover the well-known[12] exponential behavior of $\xi(T)$, that we will parameterize as

$$\frac{\xi}{\xi_0} = \frac{1}{A} e^{b/\sqrt{t}}, \quad t \to 0, \tag{12}$$

where b and A are constants of order 1.

It should be emphasized that if the bare superfluid stiffness were a constant independent of the temperature Eq. (12) would be valid until $t\sim\mathcal{O}(1)$, i.e., essentially at all relevant temperatures above $T_{\rm BKT}$. However, what limits in a crucial way the applicability of the above approximation is the temperature dependence of $J(T)$ due to quasiparticle excitations, which lead to the vanishing of $J(T)$ at the GL temperature T_c. In a s-wave BCS superconductor a good approximation for $J(T)$ is

$$\frac{J(T)}{J_0} = \left[\frac{\Delta(T)}{\Delta_0}\right]^2, \quad \Delta(T) = \Delta_0 \tanh\left(\frac{\pi}{2}\sqrt{\frac{T_c}{T}-1}\right).$$

For any practical purpose, to determine t_c what matters is only the behavior of $J(T)$ near T_c, that according to the above equation is linear,

$$J(T) \approx J_0 \frac{\pi^2}{4}\left(1-\frac{T}{T_c}\right), \quad T \approx T_c. \tag{13}$$

By neglecting the normalization of J_s with respect to J due to vortices already below $T_{\rm BKT}$, one can approximately estimate the BKT temperature by the condition (10) $J(T_{\rm BKT}) = 2T_{\rm BKT}/\pi$, so that one gets

$$t_c \approx \frac{8T_c}{\pi^3 J_0}. \tag{14}$$

Thus, one sees that the larger is J_0/T_c the smaller is the interval t_c. In a thin film of conventional superconductors[18–20] the typical mean-field temperatures are of order of few K while J_0 can be as large as the Fermi energy if $n_s^{2D}(0)$ coincides with the electron density, as it is the case in clean superconductors.[29] Indeed, Eq. (5) gives

$$J = 22.1 n_s^{2D}[10^{13}\ \text{cm}^{-2}]\ \text{K} = 0.62\frac{d[\text{A}]}{\lambda^2[(\mu\text{m})^2]}\ \text{K}. \tag{15}$$

With d of order of few nanometers and $\lambda(0)\sim 100$ Å, as it is the case for conventional clean superconductors,[29] one has J_0 of order of 10^3 K so that t_c would be of order of 10^{-3}, and then the BKT transition would be essentially indistinguishable from the mean-field T_c. However, in dirty films of superconductors J_0 can be substantially reduced with respect to the clean case[18–20] so that t_c assumes usually values around $0.05 \lesssim t_c \lesssim 0.5$, and $T_{\rm BKT}$ is sufficiently far from T_c to be observable.

From the point of view of the temperature dependence of the correlation length one sees that as one moves away from $T_{\rm BKT}$ toward T_c the decrease in $\mu(T)$ near T_c makes the increase in the vortex fugacity under RG flow very fast so that the RG superfluid density vanishes for $a(\bar{\ell})\simeq\xi_0$ and ξ scales as the unrenormalized vortex fugacity,[24] i.e.,

$$\xi = \xi_0 e^{-\beta\mu(T)/2}, \quad t \to t_c. \tag{16}$$

Since $\mu(T)\to 0$ as $T\to T_c$, it follows that $\xi=\xi_0$ at $T=T_c$. Notice that strictly speaking one could assume that also the vortex-lattice spacing ξ_0 increases as $T\to T_c$ as the mean-field correlation length. However, this just signals that as T_c is approached the phase-modulus separation that justifies the BKT approach to the fluctuations fails because the two degrees of freedom cannot be separated anymore in a controlled way. One expects that $\xi(T)$ interpolates in a continuous way between the BKT regime [Eq. (12)] and the GL regime, as we shall see below. Anyway, what is crucial to realize is that the approximate form Eq. (12) is limited to a regime $t \ll t_c$ by the existence itself of a fluctuating GL regime at higher temperatures.

As it is well known,[12] in an infinite system the BKT correlation length remains infinite anywhere below $T_{\rm BKT}$. This signals the fact that in 2D no symmetry breaking can occur so that the SC correlations decay to zero at large distance with a nonuniversal power-law decay, instead of the exponential decay to a finite value that one has in higher dimensions. From the point of view of the definition of ξ used above in terms of the scale ℓ_s where $J_s(\ell)$ vanishes, since J_s is always finite below $T_{\rm BKT}$ then $\xi=\infty$ in the SC phase. However, in any real system the RG scaling must be stopped at a certain scale $\ell_{\max}$,

$$\ell_{\max} \equiv \log\frac{L}{\xi_0}, \tag{17}$$

where L is the maximum physical scale accessible in the system (such as the system size or the size of the homogeneous domains, see Sec. IV). As a consequence, the divergence itself of the correlation length is cut off below the temperature where $\ell_s=\ell_{\max}$ so that the correlation length starts deviating from Eq. (12), and finite-size effects dominate. Sufficiently below $T_{\rm BKT}$, $K(\ell)$ is substantially unrenormalized with respect to the initial value $K(0)$ (Ref. 14) so that we can estimate finite-size effects by integrating the RG equation for g using $K(\ell)\approx K(0)$. Thus, $g_u(\ell)=g_u(0)e^{(2-K)\ell}$ from Eq. (9) and the correlation length behaves as

$$\frac{\xi}{\xi_0} = e^{\beta\mu(T)/2}\left(\frac{L}{\xi_0}\right)^{K(T)/2}, \quad T \lesssim T_{\rm BKT}. \tag{18}$$

We notice that, following Eq. (13), near T_c, $K(T)$ can be approximated as $K(T)=2(1-t/t_c)$. As a consequence, from Eq. (18) we see that the size-limited ξ grows below $T_{\rm BKT}$ as

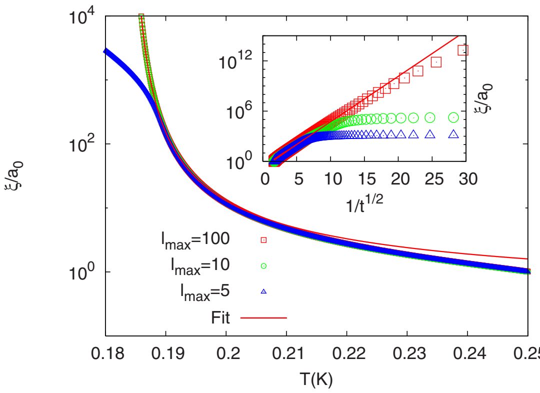

FIG. 2. (Color online) Temperature dependence of the correlation length within KT theory, obtained by numerical integration of the RG Eqs. (8) and (9) with the definition Eq. (11), where $\overline{\ell}$ is determined either by the vanishing of the superfluid stiffness or by the finite system size $\ell_{\max}$, see Eq. (17). Here $T_c = J_0 = 0.25$ K and $\tilde{\mu} = \tilde{\mu}_{XY}$. The solid line represents the fit done using Eq. (12) with $A=5$ and $b=1.25$. Since $T_{BKT}=0.18$ the b value obtained by the fit is consistent with Eq. (21). Inset: same data plotted as a function of $1/\sqrt{t}$, where t is the reduced temperature defined in Eq. (3). Notice that the curves for lower $\ell_{\max}$ deviate from the infinite-length-scale limit [Eq. (12)] at higher temperatures.

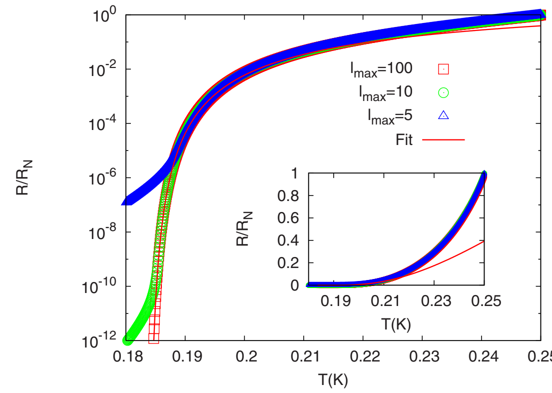

FIG. 3. (Color online) Fluctuation resistivity in KT theory as a function of temperature for several values of the system size. The same set of parameters of Fig. 2 has been used. Inset: same curves but on a linear scale, where no appreciable difference can be observed for the different system sizes.

$$\frac{\xi}{\xi_0} \sim \left(\frac{L}{\xi_0}\right)^{|t|/t_c}. \tag{19}$$

Thus, if one considers two cases with the same L and T_{BKT} but different t_c the finite-size effects are less pronounced in the case with smaller t_c because ξ is still very large below T_{BKT}.

As an example of the correlation-length temperature dependence we show in Fig. 2 the numerically integrated ξ at various values of the scale L, along with the fit of Eq. (12). Here we consider the case $J_0 = T_c = 0.25$ K so that a relatively large interval $t_c \simeq 0.36$ is obtained, to better shown the BKT-to-GL crossover. From the inset of Fig. 2 we observe that to correctly capture the exponential scaling one must use extremely large sizes, far beyond the experimentally accessible regime. Indeed, since for conventional superconductors usually $\xi_0 \simeq 100$ Å and $L \simeq 1$ cm, then $L = 10^6 \xi_0$ and $\ell_{\max} = 12$ while the exponential fit in Fig. 2 is better defined with $\ell_{\max} = 100$.

As one can see in the main panel of Fig. 2, the exponential fit [Eq. (12)] deviates slightly from the RG correlation length as one moves to temperatures higher than T_{BKT}. This qualitative difference is also reflected by the temperature dependence of the fluctuations resistivity, that can be calculated according to Eq. (2), by using either the RG or the approximate form (12) of the correlation length. The result for the same set of parameters used in Fig. 2 is shown in Fig. 3; as shown in the inset, on a linear scale R has always an upward curvature, with no appreciable differences for the different L values. Such a difference becomes instead evident on a logarithmic scale: as $T \to T_{BKT}$, R/R_N deviates from the infinite-size limit, exactly in the same fashion observed in experiments in thin films (see, for example, Fig. 9 of Ref. 19). This scaling can be probed by small magnetic fields; indeed, a small field has the effect to cut off the RG at a scale $L_B \propto 1/B$. The curves shown in Fig. 11 of Ref. 19 for the fluctuation resistivity at several (small) values of the magnetic field reproduce exactly the behavior observed in Fig. 3.

Due to the need of exceedingly large system sizes for an accurate determination of the parameters of the fit [Eq. (12)], an *a priori* estimate of them as a function of physical parameters would be particularly useful. To address this issue we computed ξ for several possible values of t_c and $\tilde{\mu}$, to analyze the variations in b. While different t_c values naturally occur in different systems, the choice of $\tilde{\mu}$ is somehow still an open problem. As we mentioned above, within the XY model (that is one of the possible models for describing phase fluctuations in a superconductor) $\tilde{\mu}_{XY} = \pi^2/2$; this value naturally arises from the mapping of the discrete XY model on the continuum Coulomb-gas model,[32] and it takes into account the fluctuations at a scale on the order of the lattice spacing. In a BCS superconductor one could instead fix the value of the vortex-core energy by computing exactly the energy per unit length of a vortex line,[29]

$$I = \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \left[\log\frac{\lambda}{\xi_0} + \epsilon\right] \equiv \pi J \left[\log\frac{\lambda}{\xi_0} + \epsilon\right]$$

so that according to our definition $\tilde{\mu} = \pi\epsilon$. A precise estimate of $\epsilon \simeq 0.497$ for the vortex core in three-dimensional geometry is given in Refs. 33 and 34 so that within BCS one could eventually expect smaller values of μ,

$$\tilde{\mu}_{BCS} \simeq \frac{\pi}{2} \simeq \frac{\tilde{\mu}_{XY}}{\pi}. \tag{20}$$

Finally, we notice that recent analysis in the context of cuprate superconductor[21,24,26,27] has explored instead the possibility that $\tilde{\mu}$ is larger than $\tilde{\mu}_{XY}$. On the light of the previous observations, we considered the behavior of the correlation length for a range of values $0.5 \lesssim \tilde{\mu} \lesssim 1.2$, and we extracted the corresponding b parameter by fitting data near T_{BKT} with Eq. (12). The results are shown in Fig. 4. As one can see, within a certain degree of uncertainty, b is found to scale approximately as

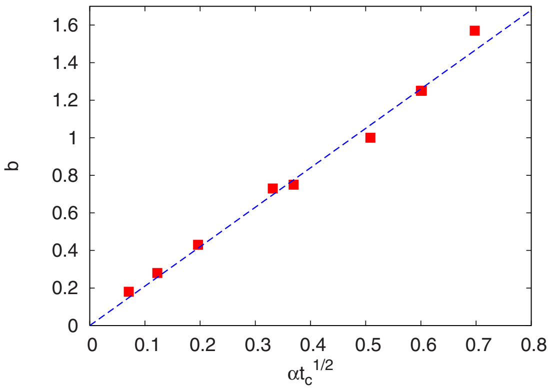

FIG. 4. (Color online) Estimate of b extracted by fitting the RG $\xi(T)$ near T_{BKT} with Eq. (12) for several values of t_c, α. The straight line is $y=2.1x$.

$$b \simeq 2\alpha\sqrt{t_c}, \tag{21}$$

where α is the scale of the vortex-core energy defined in Eq. (7). Equation (21) is the first important result of this paper. Indeed, it establishes a precise relation between the parameter b that appears in the typical exponential expression for the BKT correlation length and the distance t_c between the GL and BKT temperatures. Taking into account Eqs. (12) and (16), we also see that the formula (12) can only be used when $b/\sqrt{t}\gg 1$, which means $t \ll 4\alpha^2 t_c$. Indeed, out of this regime ξ decreases according to Eq. (16), and afterward ($T > T_c$) one enters the GL fluctuations regime. We notice also that this result agrees with the HN result (4), once one uses $b_{\mathrm{HN}}=2\alpha$.

The analysis we have made by coupling the standard BKT formula (12) with the RG analysis of the BKT transition thus allows us to get strong constraints on what the "fit parameters" that are to be used in the BKT formula can be. This will help in the following to take into account additional effects such as the ones of inhomogeneities, but specially to know if the fit to the BKT functional form corresponds to a physical fit, with reasonable parameters, or if one just "forces the fit" (see discussion in Sec. V).

IV. ROLE OF INHOMOGENEITIES

Once we established a clear framework for taking into account the exponential behavior of the correlation length and the finite-size effects we can improve the HN original interpolation formula for paraconductivity, in order to obtain a self-consistent treatment of SC fluctuations all the way below and above T_{BKT}. We propose the following interpolating formula:

$$\frac{R}{R_N} = \frac{1}{1+(\Delta\sigma/\sigma_n)} \equiv \frac{1}{1+(\xi/\xi_0)^2}, \tag{22}$$

where ξ is given by

$$\frac{\xi}{\xi_0} = e^{\beta\mu(T)/2}\left(\frac{L}{\xi_0}\right)^{\pi J(T)/2T}, \quad T \lesssim T_{\mathrm{BKT}}, \tag{23}$$

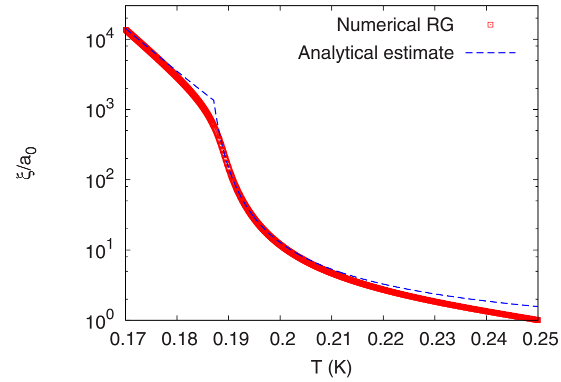

FIG. 5. (Color online) Comparison between the numerical RG correlation length defined by Eq. (11) and the approximate formulas (23) and (24). Here we used the same T_{BKT}, T_c, μ values of Fig. 2 in the case $\ell_{\max}=5$ so that $b=1.25$ and $A=5$.

$$\frac{\xi}{\xi_0} = \frac{2}{A}\sinh\frac{b}{\sqrt{t}}, \quad T \gtrsim T_{\mathrm{BKT}} \tag{24}$$

with the parameter A, b obtained by the calculated behavior of the numerical RG correlation length near the transition so that b is given by Eq. (21) and A is a number of order 1. As it is shown in Fig. 5 this is indeed a very good approximation for the numerical RG solution near and below T_{BKT}. Moreover, even though it does not capture the regime [Eq. (16)], where the estimate [Eq. (24)] is larger than the RG correlation length, it correctly reproduces the GL fluctuation regime at $T \gg T_c$, where $\Delta\sigma \sim T_c/(T-T_c)$. We stress once more that within this approach the temperature dependence of the correlation length in the whole fluctuating regime is uniquely determined by the critical temperatures T_{BKT}, T_c and the value of the vortex-core energy, that can be fixed by the comparison with the experimental resistivity data.

The main physical message of the set of Eqs. (21), (23), and (24) is that from the temperature dependence of the fluctuation resistivity between T_{BKT} and $T \gg T_c$ one can deduce interesting informations also on the microscopic parameters of the superconducting system, that determine the distance between T_c and T_{BKT}. To illustrate the application of this approach we consider the analysis of the resistivity data in superconducting heterostructures reported in Ref. 4. Here the resistive transition occurs around 0.19 K but with a relatively large tail with respect for example to what observed in 2D films of ordinary superconductors.[18–20] If we neglect the tail, by means of Eqs. (23) and (24) we can obtain the curve labeled as "Hom" in Fig. 6. As one can see, we reproduce the overall shape of the resistivity, except from the tail. Since the fit gives $b=0.19$, according to Eq. (21) we deduce that T_c and T_{BKT} almost coincide, with $T_c=0.19$ K and $T_{\mathrm{BKT}}=0.188$ K.

Let us discuss now the origin of the remaining tail of the resistivity near the transition. First of all, we notice that it cannot be due to finite-size effects, that can be treated exactly within our approach. Indeed, even using a relatively small $L=2$ μm (as suggested by the critical current, see below), with $\xi_0=70$ Å,[4] one gets $\ell_{\max}=5.6$, but due to the small t_c value the ξ from Eq. (23) is still very large below

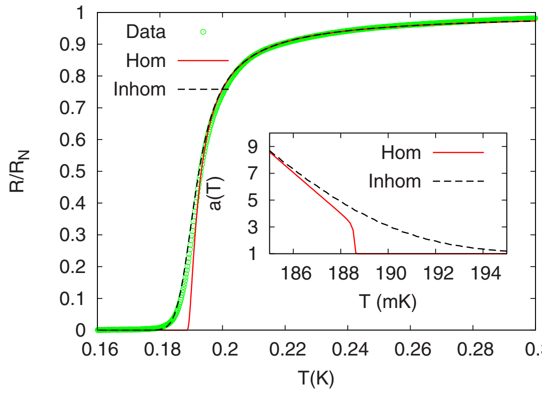

FIG. 6. (Color online) Comparison between the resistivity of the heterostructure measured in Ref. 4 and the resistivity obtained with the interpolating formula (22). The curve labeled Hom refers to the case of a single local J value while the curve "Inhom" refers to the resistivity obtained by sample average over the local distribution [Eq. (25)] of superfluid density. Inset: coefficient $a(T)=1+\pi J_s(T)/T$ of the I-V characteristic in the two cases.

$T_{\rm BKT}$ so the finite-size effects by themselves are *not responsible* for the observed tail. On the other hand, from Fig. 3 we notice that a tail with upward curvature is typical of the fluctuation resistivity near the $T_{\rm BKT}$ transition. However since t_c in this case is extremely small, the tail cannot be even resolved in the scale of Fig. 6. On the contrary, if the transition *itself* is broadened one could expect to enhance the BKT tail and to reproduce the experimental data.

On the light of this observation, we suggest that the resistive tail can be attributed to an inhomogeneous spatial distribution of the local superfluid, to be ascribed to an intrinsic inhomogeneous density distribution in these systems. In analogy with the analysis of inhomogeneity on the superfluid density performed in Ref. 21, we shall assume for simplicity a Gaussian profile,

$$P(J) = \frac{1}{\sqrt{2\pi}\delta}\exp\left[\frac{-(J-\bar{J})^2}{2\delta^2}\right], \tag{25}$$

where $\bar{J}$ is the J_0 value determined by the above fit for the homogeneous case. To each J corresponds a mean-field temperature $T_{mf}=T_c(J/\bar{J})$, and accordingly a given $T_{\rm BKT}$ value. While far from $T_{\rm BKT}$ such an inhomogeneity is harmless, near $T_{\rm BKT}$ it will give important effects, once we average over different patches having different transition temperatures. To compute the effect of the sample inhomogeneity on the resistivity we go back to Eq. (2), that defines the resistivity due to vortices. We notice that the main quantity that enters the theory is the vortex density; indeed, the correlation length defined by Eq. (2) is just a different way to express the vortex density. Thus, in the presence of the inhomogeneity [Eq. (25)], we must average the vortex-density values $n_F(J)$ obtained in each patch with a given local J value; in this way, even below $T_{\rm BKT}$ there will be patches of the system where n_F is finite because $J<\bar{J}$ and consequently the local $T_{\rm BKT}$ is smaller than the average one. As a consequence, the BKT tail gets enhanced, in excellent agreement with the experiments (with $\delta=0.02\bar{J}$), see Fig. 6.

A second effect of the inhomogeneity that emerges in the experiments of Ref. 4 is the lack of the universal jump of the superfluid density near J_s. In Ref. 4, J_s has been indirectly measured via the I-V characteristics in the nonohmic regime [see discussion above Eq. (28)], given within the KT theory by[13,17,35]

$$V = I^{1+\pi J_s(T)/T} = I^{a(T)}. \tag{26}$$

Thanks to the universal relation (10), right below the transition $\pi J_s(T_{\rm BKT})/T_{\rm BKT}=2$ so that the coefficient $a=3$. On the other hand, in the infinite-size homogeneous case $J_s(T)$ jumps discontinuously to zero right above $T_{\rm BKT}$ so that the coefficient $a(T)$ is expected to jump discontinuously to 1. Even though finite-size effects remove the jump and lead to a rapid downturn (see inset of Fig. 6), the measured temperature dependence of $a(T)$ is still much smoother. This effect can be explained by the presence of inhomogeneity, that leads to a smooth downturn of J_s near $T_{\rm BKT}$.[21]

As a further check of the correctness of our analysis of the data, we discuss the implication of the interpolated GL +BKT fit on the estimated value of the superfluid-density value at $T=0$. From the value of $t_c\approx 8T_c/(\pi^3 J_0)=0.0106$ obtained by the fit, it follows [see Eq. (14)] that $J_0=7$ K. According to Eq. (5) above, J_0 is in turn controlled by the zero-temperature value of the density of superfluid electrons. While in a clean superconductor $n_s^{2D}=n^{2D}$ at $T=0$, where n^{2D} is the sheet carrier density of the sample, in the dirty case only a fraction on the order of ℓ_d/ξ_0 of the electron density condenses into the superfluid fraction at $T=0$, where $\ell_d=v_F\tau$ is the mean-free path, with v_F Fermi velocity and τ scattering time. Since the BCS correlation length at $T=0$ is $\xi_0=\hbar v_F/\pi\Delta(0)$, with Δ superconducting gap, and τ can be determined by the sheet resistance $R_\square=n^{2D}e^2\tau/m^*$ (here $\square$ denotes the sheet area), we obtain an estimate of the disorder-reduced superfluid density as

$$k_BJ_0 = \frac{\hbar^2 n_s^{2D}}{4m^*} \approx \frac{\hbar^2 n^{2D}}{4m^*}\frac{\pi\Delta(0)\ell}{\hbar v_F} = \frac{\hbar}{e^2}\frac{n^{2D}e^2\tau}{m^*}\frac{\pi\Delta(0)}{4} = \frac{R_c}{R_\square}\frac{\pi\Delta(0)}{4}. \tag{27}$$

Using the universal value $R_c=\hbar/e^2=4114\ \Omega/\square$, the measured normal-state resistance $R\simeq 390\ \Omega/\square$, and the BCS estimate $\Delta(0)=1.76T_c$ with $T_c=0.19$ K, we obtain $J_0\simeq 3$ K, which is in excellent agreement with the value $J_0=7$ K obtained by the fit, taking into account the approximate estimate of the condensate fraction used in Eq. (27) above.

The crucial role of inhomogeneities in controlling the tails of the resistive transition is made even more evident by extending the above analysis to a second example of superconducting heterostructures measured in Ref. 5, where the normal-state resistance is higher so that inhomogeneous effects could be expected to be stronger. We shall consider the case of zero gate voltage in Ref. 5, i.e., the as-grown sample without the field-induced doping. The data are shown in Fig. 7 along with our proposed fit ($T_c=0.269$ K, $T_{\rm BKT}=0.25$ K,

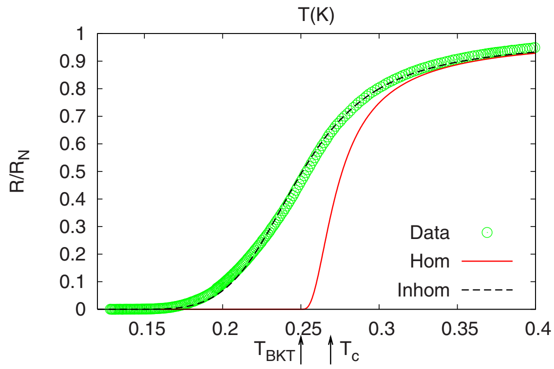

FIG. 7. (Color online) Comparison between the experimental data in the unbiased sample ($V=0$) of Ref. 5 and the resistivity obtained with the interpolating formula (22), with (Inhom) and without (Hom) inhomogeneity. The parameter values are given in the text. The two arrows mark $T_{\rm BKT}$ and T_c obtained with the fit in the homogeneous case.

$b=0.63$, and $J_0=1.05$ K). Notice that a larger b is consistent with a larger t_c, that in turn can be attributed to a larger disorder in this second case. Indeed, in this sample $R_\square=2400$ $\Omega/\square$, and since $n^{2D}=4.5\times10^{13}$ cm^{-2} using Eq. (27) we obtain a consistently smaller estimate of $J_0\approx 0.6$ K, which is exactly the result of the fit (a factor of ten smaller than the previous case). Moreover, the inhomogeneity is also larger, with $\delta=0.14\bar{J}$, and this explains the very large tail of the transition.

As we emphasized above, finite-size effects alone cannot account for the broadening of the transition. However, one could convert in a hand-waving way the inhomogeneities effects that we discuss into some form of typical size for "homogeneous" grains in the system. Let us emphasize that this is just a way to visualize the role of the inhomogeneities and that we do not assume any sharp granular structure here, see Fig. 8. The inhomogeneity distribution [Eq. (25)] and, in particular, the broadening of J values, δ, could be converted into a characteristic size $L_{\rm hom}$ much smaller than the true system size. An indication in this sense is provided by the analysis of the I-V characteristics reported in Ref. 4. Indeed,

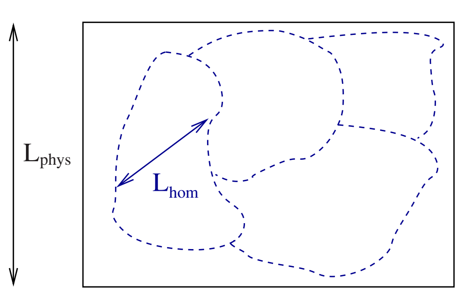

FIG. 8. (Color online) Schematic sketch of the mesoscopic structure of a 2D film that can account for the inhomogeneity effects discussed in this work. Even though the system does not have a true granular structure the homogeneous regions will have a typical size $L_{\rm hom}$ smaller than the physical size $L_{\rm phys}$ of the sample. This explains why $L\equiv L_{\rm hom}<L_{\rm phys}$ in Eq. (29), giving rise to a critical current for linear-to-nonlinear characteristic larger than expected for the homogeneous case.

within BKT physics the I-V characteristic is ohmic at low current and nonohmic when the external current is sufficiently high to dissociate vortex-antivortex pairs. Below $T_{\rm BKT}$, the critical current I^* above which current-induced free vortices lead to the anomalous power-law dependence described by Eq. (26) above, is in turn related to a characteristic length scale L, where finite-size effects become relevant[13,17,35] so that

$$I^* = 2\pi J_s \frac{c}{\Phi_0}\frac{L_{\rm phys}}{L}, \tag{28}$$

where $L_{\rm phys}$ is the physical dimension of the sample. By using the approximate relation between J_s and $T_{\rm BKT}$, one can estimate for the sample of Ref. 4 that

$$I^* \simeq 4K_B T_{\rm BKT}\frac{c}{\Phi_0}\frac{L_{\rm phys}}{L} = \frac{L_{\rm phys}}{L}0.5\times10^{-8}\ \text{A}. \tag{29}$$

Since experimentally the critical current is of order of 0.5 $\times10^{-6}$ A and $L_{\rm phys}\simeq 0.2$ mm, we deduce that $L\sim 2$ μm. This can be in turn identified with the size $L_{\rm hom}$ of the homogeneous domains so that $\ell_{\rm max}=5.6$, as we used in our calculations. Notice that Eq. (28) accounts also for the temperature dependence of $I^*(T)$ observed in the experiments; indeed, as we have seen $J_s(T)$ increases rapidly below $T_{\rm BKT}$ [see the behavior of the $a(T)$ exponent in Fig. 5], leading to a very fast increase in the critical current below the $T_{\rm BKT}$ transition. At temperatures well below $T_{\rm BKT}$ one should further account in the analysis of the I-V characteristic for the nucleation of virtual vortex-antivortex pairs, formed by a single vortex and its virtual image due to the edge currents in a finite-size system, as pointed out recently in Ref. 36.

V. DISCUSSION ON THE ANALYSIS OF THE EXPERIMENTAL DATA

In the previous section we performed a systematic analysis of the resistivity data from Refs. 4 and 5. Our analysis is based on the regular BKT transition and its connection with Gaussian fluctuations leads to the conclusion that the BKT transition occurs *very near* (within few percent) of the mean-field GL transition temperature T_c. In this case the BKT fluctuation regime is restricted to a small range t_c of reduced temperatures near $T_{\rm BKT}$, where the correlation length [Eq. (24)] has the exponential behavior [Eq. (12)], with an exponential divergence near $T_{\rm BKT}$ that is controlled by the same parameter b [Eq. (21)] that measures the distance from the GL transition T_c. At temperatures near and below $T_{\rm BKT}$ inhomogeneities are responsible for the tail in the resistive transition.

A quite different way to interpret the data was followed instead in Refs. 4, 5, and 28, based essentially on the idea that T_c is far larger than $T_{\rm BKT}$ so that the *whole* fluctuation regime should be described via the standard BKT approach, see Fig. 1. For the sake of clarity in the following we will refer to our approach as "BKT-GL" [Fig. 1(a)], and to the one proposed in Refs. 4, 5, and 28 as "BKT-only" [Fig. 1(b)]. In the latter case, the expression (12) for the correlation length should be valid in *all* the temperature regime where

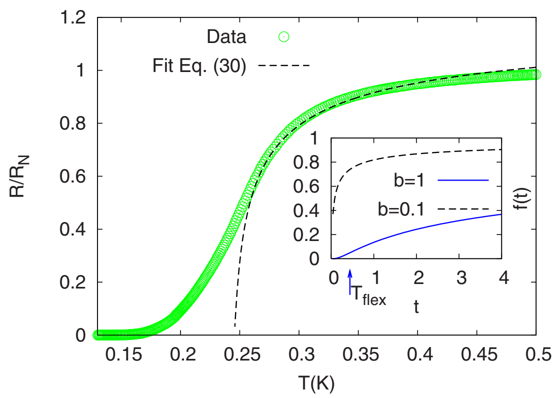

FIG. 9. (Color online) Example of application of the BKT fit proposed in Ref. 28 for the same data shown in Fig. 7. The theoretical curve is Eq. (30) with $T_{\rm BKT}=0.245$ K and $b=0.106$. Inset: temperature dependence of the function $f(t)=\exp(-2b/\sqrt{t})$ for two b values. For $b=1$ one can clearly distinguish on this scale a first regime with upward curvature, up to the temperature T_{flex} given by Eq. (31), followed by a regime with downward curvature. For $b=0.1$ only the second regime is visible on this scale.

R/R_N deviates from 1 and b is a number of order 1, to assure that T_c is enough far from $T_{\rm BKT}$. As a consequence, the *whole* resistivity [Eq. (2)] is given at all temperature above $T_{\rm BKT}$ by

$$\frac{R}{R_N}=A^2e^{-2b\sqrt{T_{\rm BKT}/(T-T_{\rm BKT})}}=A^2f(t), \tag{30}$$

where $f(t)=\exp(-2b/\sqrt{t})$ and t is the reduced temperature [Eq. (3)]. An example of application of Eq. (30) to the same resistivity data analyzed in Fig. 7 above is presented in Fig. 9. Here we used Eq. (30) with $T_{\rm BKT}=0.245$ K and considered for the moment b as a completely *free* parameter. In this case, a reasonable fit of the data can be obtained by using $b=0.106$. Such a small value of b can be understood by looking at the behavior of the function $f(t)$ shown in the inset of Fig. 9. As one can notice, if one extend $f(t)$ at arbitrary t values above $T_{\rm BKT}$ it actually saturates to a constant value. The crossover from the low-T regime where $f(t)$ displays an upward curvature to the one where it displays an downward curvature can be estimated by the flex of this curve, that is found at

$$T_{flex}=T_{\rm BKT}[1+0.44b^2]. \tag{31}$$

As a consequence, if b is very small the function $f(t)$ saturates very rapidly and one can fit the experimental data with the expression (30) by assuming a very small b value. However, this procedure is totally inconsistent with the fact that within BKT theory b is not a free parameter, as we discussed in the previous sections but it depends, through Eq. (21), on the distance t_c between the BKT and GL temperatures. In particular, the fit presented in Fig. 9 gives $b=0.1$, that would imply a $t_c=0.0025$, in clear contradiction with the *a priori* assumption that T_c is so much larger than $T_{\rm BKT}$ than the whole fluctuation regime has BKT character. Moreover, a smaller b value would also imply that J_0 is very large ($J_0 \sim 26$ K), that is again inconsistent with the estimate [Eq. (27)] based on the normal-state resistivity value (i.e., $J_0 \sim 7$ K). We note, in particular, that this procedure leads to a value for the parameter b that is about *one order of magnitude smaller* than what we obtained in our case (see Fig. 7), where $b=0.6$. Indeed, in the BKT-GL approach most of the fluctuation regime is accounted by GL fluctuations so that Eq. (30) is actually valid only in the limited range of temperatures $T \ll T_{flex}$, where $f(t)$ displays an upward curvature.

An incorrect use of the BKT formulas can lead to quite unphysical values for the b parameter, as it is the case in our opinion for Ref. 28. Moreover, independently of the question of the distance between $T_{\rm BKT}$ and T_c, a second drawback of this BKT-only analysis is that it attributes the tail of the transition, that is the real signature of BKT physics, to unrealistic finite-size effect. Indeed, in Ref. 28 it has been proposed that the deviation from the best fit obtained with Eq. (30) and the experimental data occurs when ξ is cut off by finite-size effects. As we have seen above, finite-size effects become relevant near $T_{\rm BKT}$ when $\xi \sim L$, see Eq. (18). Thus, since $R/R_N=(\xi_0/\xi)^2$ according to Eq. (2), in Ref. 28 it has been argued that the fit with Eq. (30) deviates from the experimental data when $\xi(T)=L$, with $T>T_{\rm BKT}$. By applying this idea, since from Fig. 9 it follows that the fit deviates from the data already at $R/R_N \sim 0.5$, one would obtain $L \sim 2\xi_0 \sim 100$ Å. This is an unrealistic small number for the size of the homogeneous domains because it would not allow at all the formation of coherent vortex structures (and then the observation of their signatures) on a distance as small as only twice the lattice spacing for phase fluctuations (i.e., ξ_0). In our approach instead the deviation of the experimental data from the homogeneous-case fit is due to inhomogeneity while finite-size effects alone cannot account for it.

The only way to possibly justify the BKT-only analysis of the data would be if for some reason the constraints on the b parameter that we establish in our paper for a conventional BKT transition would not be obeyed, namely, if the transition is of a different nature than the conventional BKT transition. What could be such an alternative is unclear from a theoretical point of view. A proposal in this sense has been formulated in Ref. 4, where it has been argued that in these systems one can estimate the initial value of the vortex fugacity in the RG Eqs. (8) and (9) to be quite large, invalidating the applicability itself of the perturbative RG.[13,37] In this case, following the analysis of Ref. 37, the BKT transition has a qualitative different character since it refers to the melting of the 2D lattice of vortices formed at high vortex density. Such a scenario can provide, for example, an alternative interpretation to the high values of the critical current I^* [Eq. (28)] mentioned above (see Supplementary Information of Ref. 4). This is thus an interesting proposal but it leads to a BKT-only analysis of the data that still suffers from two main problems: (a) no theoretical work has been devoted so far to explore the signatures of such a melting transition, if it exists, on the SC fluctuations contribution to the resistivity. In particular, it is not at all obvious that such a scenario would lead to a different b parameter so that the small b value obtained by the application of Eq. (30) at all temperatures above $T_{\rm BKT}$ would be meaningful. (b) Recent numerical simulations on the vortex liquid do not confirm the existence of this crystal phase[38] even at high vortex fugacity and only standard BKT vortex phase seems to be present so that it is not even clear whether such a melting transition would exist. Thus, more theoretical and experimental work is certainly needed to test this possibility and to decide which one of the interpretations, the BKT-GL approach or the BKT-only based on the idea of vortex-lattice melting, is indeed the correct explanation of the existing data. For example, further measurements of the size dependence of the critical current I^* or the direct measurement (via two-coils mutual inductance experiments) of the temperature dependence of the superfluid density can provide us with additional information on the system, to be tested against the different theoretical proposals.

VI. CONCLUSIONS

In the present work we investigated the role of finite-size effects and inhomogeneity of the BKT transition in quasi-2D SC systems. By analytical and numerical analysis of the RG equations for the BKT transitions we determined in a precise way how the properties of the system control the behavior of the correlation length above and below the transition. By interpolating the BKT behavior with the GL fluctuations we propose an unified scheme to determine both the mean-field transition temperature T_c and the BKT one $T_{\rm BKT}$ from a fit of the resistivity data. In our approach, those that are usually treated as free parameters in the BKT fitting formula are implicitly determined by the these temperature scales, and allows us to identify the effects that must be attributed to the finite system size and inhomogeneity. The direct comparison with recent experiments in SC heterostructures suggests that spatial inhomogeneity, whose role we discussed already in the context of ultrathin films of high-T_c superconductors,[21] is a common ingredient to these systems as well. Our analysis allows us to estimate also the superfluid-density content of these unconventional SC interfaces, whose direct experimental determination is not yet available. Interestingly, in Ref. 5 it has been shown that by decreasing the carrier density by field-effect one can induce in such heterostructures the transition from a SC to a metallic and then insulating state. An interesting open issue is the role played by inhomogeneity in such a crossover, for example, one could expect that as $J_0(T_c)$ decreases and δ increases one reaches the percolative threshold below which the system cannot sustain anymore superfluid currents, in analogy with the behavior of diluted XY models.[39] This issue can have profound consequences on the nature of the quantum critical point inferred in Ref. 5, and certainly deserves further investigation.

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